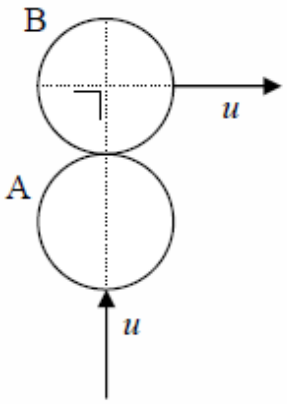
2006 – IMPACTS AND COLLISIONS QUESTION

5. **(a)** A smooth sphere P, of mass 3 kg, moving with speed 6 m/s, collides directly with a smooth sphere Q, of mass 5 kg, which is moving in the same direction with speed 2 m/s. The coefficient of restitution for the collision is e.

(i) Find, in terms of e, the speed of each sphere after the collision.

(ii) If the loss of kinetic energy due to the collision is $k(1-e^2)$, find the value of k.

(b) A smooth sphere A moving with speed u, collides with an identical smooth sphere B which is moving in a perpendicular direction with the same speed u.



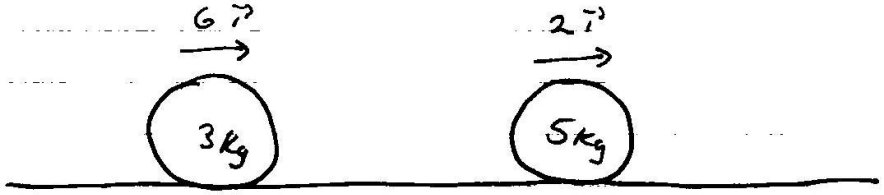
The line of centres at the instant of impact is perpendicular to the direction of motion of sphere B.

The coefficient of restitution between the spheres is e.

(i) Find, in terms of e, the speed of each sphere after impact and hence, or otherwise, show that it is not possible for the two spheres to have the same speed after impact.

(ii) Prove that $\tan\theta = \left(\frac{1+e}{2}\right)$, where θ is the angle through which sphere B is turned as a result of the impact.

2006
Q5
(c)



(i)

BEFORE	MASS	AFTER
$6\vec{i}$	3 kg	p
$2\vec{i}$	5 kg	q

Con. of Mom: $3(6) + 5(2) = 3(p) + 5(q)$

$18 + 10 = 3p + 5q$

$$\boxed{28 = 3p + 5q}$$

Coeff. of Res: $\dfrac{p - q}{6 - 2} = -e$

$$\boxed{p - q = -4e}$$

SOLVING:

$3p + 5q = 28$

$5p - 5q = -20e$

$8p = 28 - 20e$

$2p = 7 - 5e$

$$\boxed{p = \frac{7 - 5e}{2}}$$

$p - q = -4e$

$p + 4e = q$

$\dfrac{7 - 5e}{2} + 4e = q \quad (\times 2)$

$7 - 5e + 8e = 2q$

$7 + 3e = 2q$

$$\boxed{\frac{7 + 3e}{2} = q}$$

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Q5 (a)

(ii) K.E. Before : $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$

$= \frac{1}{2}(3)(6)^2 + \frac{1}{2}(5)(2)^2$

$= 54 + 10 = 64\ J$

K.E. After : $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$= \frac{1}{2}(3)\left[\frac{7-5e}{2}\right]^2 + \frac{1}{2}(5)\left[\frac{7+3e}{2}\right]^2$

$= \frac{3}{2}\left[\frac{49-70e+25e^2}{4}\right] + \frac{5}{2}\left[\frac{49+42e+9e^2}{4}\right]$

$= \frac{147-210e+75e^2}{8} + \frac{245+210e+45e^2}{8}$

$= \frac{392+120e^2}{8}$

$= (49+15e^2)\ J$

Loss in K.E. $= 64 - (49+15e^2)$

$= 64 - 49 - 15e^2$

$= 15 - 15e^2$

$= 15(1-e^2)$

So

$K(1-e^2) = 15(1-e^2)$ ... $\therefore \boxed{K=15}$

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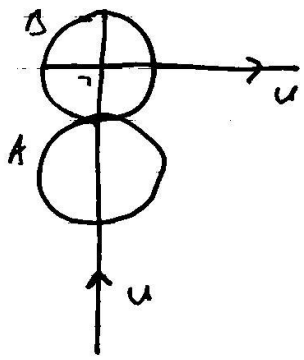
Q5
(b)



* COLLISION IS ALONG $\vec{j}$-AXIS, SO $\vec{i}$ COMPONENTS DO NOT CHANGE.

*

	BEFORE	MASS	AFTER
A)	$0\vec{i} + u\vec{j}$	m	$0\vec{i} + p\vec{j}$
B)	$u\vec{i} + 0\vec{j}$	m	$u\vec{i} + q\vec{j}$

Conservation of Momentum: $m(0) + m(u) = m(p) + m(q)$

$$\boxed{u = p + q}$$

Co-efficient of Restitution: $\dfrac{p - q}{u - 0} = -e.$

$$\boxed{p - q = -eu}$$

Solving:

$$p + q = u$$
$$\underline{p - q = -eu}$$
$$2p = u - eu$$
$$\boxed{p = \frac{u - eu}{2}}$$

$$p + q = u$$
$$\frac{u - eu}{2} + q = u \quad (\times 2)$$
$$u - eu + 2q = 2u$$
$$2q = u + eu$$
$$\boxed{q = \frac{u + eu}{2}}$$

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Q5 (b) Speed of A after: $\sqrt{(0)^2 + \left[\frac{u - eu}{2}\right]^2}$

Speed of A: $\sqrt{0 + \frac{u^2 - 2eu^2 + e^2u^2}{4}}$

Speed of A: $\sqrt{\frac{u^2 - 2eu^2 + e^2u^2}{4}}$

Speed of B: $\sqrt{u^2 + \left(\frac{u + eu}{2}\right)^2}$

Speed of B: $\sqrt{u^2 + \frac{u^2 + 2eu^2 + e^2u^2}{4}}$

Speed of B: $\sqrt{\frac{4u^2 + u^2 + 2eu^2 + e^2u^2}{4}}$

Speed of B: $\sqrt{\frac{5u^2 + 2eu^2 + e^2u^2}{4}}$

IF

Speed of A = Speed of B

$$\sqrt{\frac{u^2 - 2eu^2 + e^2u^2}{4}} = \sqrt{\frac{5u^2 + 2eu^2 + e^2u^2}{4}}$$

$$u^2 - 2eu^2 + e^2u^2 = 5u^2 + 2eu^2 + e^2u^2$$

$$-4u^2 = 4eu^2$$

$$-1 = e \quad \text{... NOT TRUE!}$$

so Speed of A $\neq$ Speed of B.

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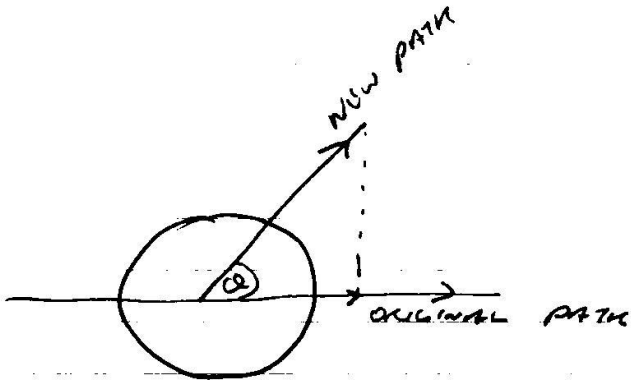
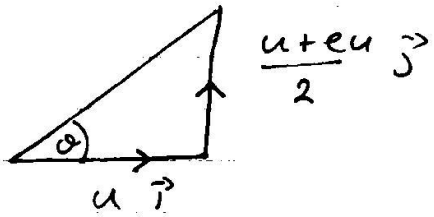
Q5

(b) Velocity of B:





$\frac{u+eu}{2}\vec{j}$

$$\tan\theta = \frac{\vec{j}}{\vec{i}}$$

$$\tan\theta = \frac{\frac{u+eu}{2}}{u}$$

$$\tan\theta = \frac{1+e}{2}$$